

Class-XII

Mathematics(041)





SECTION - A

Question : 1

$$\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$$

~~$$\vec{b} + \vec{c} =$$~~

$$\vec{b} + \vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$$

projection of $(\vec{b} + \vec{c})$ on $\vec{a} =$

$$\frac{\vec{a} \cdot (\vec{b} + \vec{c})}{|\vec{a}|}$$

~~$$|\vec{a}|$$~~

$$= \frac{(2\hat{i} - 2\hat{j} + \hat{k}) \cdot (3\hat{i} + \hat{j} + 2\hat{k})}{\sqrt{2^2 + (-2)^2 + 1^2}}$$

$$= \frac{(2 \times 3) + (-2 \times 1) + (1 \times 2)}{\sqrt{9}}$$

$$= \frac{6 - 2 + 2}{\sqrt{9}}$$

$$= \frac{6 - 2 + 2}{3}$$

$$= \frac{6}{3}$$

$$= 2$$



OR

$$= \frac{6}{3}$$

2 units

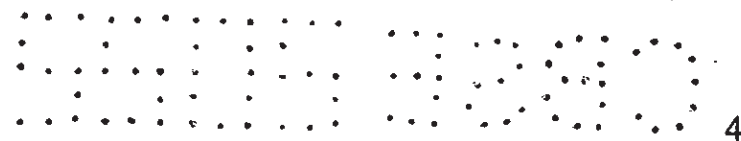
Answer: Projection of $(\vec{b} + \vec{c})$ on $\vec{a}$ is 2 units.

Question: 2

$$\log \left(\frac{dy}{dx} \right) = ax + by$$

$$\Rightarrow \frac{dy}{dx} = e^{ax+by}$$

$$\Rightarrow \frac{dy}{dx} = e^{ax} \cdot e^{by} \quad [e^{a+b} = e^a \cdot e^b]$$



$$\Rightarrow \frac{dy}{e^{by}} = e^{ax} dx$$

$$\Rightarrow e^{-by} dy = e^{ax} dx$$

On integrating both sides.

$$\int e^{-by} dy = \int e^{ax} dx$$

$$\Rightarrow \frac{-e^{-by}}{b} = \frac{e^{ax}}{a} + c$$

$$\frac{e^{ax}}{a} + \frac{e^{-by}}{b} = c' \quad [c' = -c]$$

where c & c' are constants.

Answer

$$\frac{e^{ax}}{a} + \frac{e^{-by}}{b} = e$$



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Question : 3

No. of spades = 13

Total no. of cards = 52

x denote number of spades

$$P(x=0) = P(\text{not spade}) \times P(\text{not spade})$$

$$= \frac{39}{52} \times \frac{39}{52}$$

$$= \frac{9}{16}$$

$$P(x=1) = P(\text{spade}) \times P(\text{not spade}) + P(\text{not spade}) \times P(\text{spade})$$

$$= \frac{13}{52} \times \frac{39}{52} + \frac{39}{52} \times \frac{13}{52}$$

$$= \frac{6}{16}$$



$$P(X=2) = P(\text{spade}) \times P(\text{spade})$$

$$= \frac{13}{52} \times \frac{13}{52}$$

$$= \frac{1}{16}$$

Answer

No. of spades (x)	Probability
$x=0$	$9/16$
$x=1$	$6/16 = 3/8$
$x=2$	$1/16$



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Question : 4

(option-2)

$$P(A \text{ hits}) = \frac{1}{3} \quad [P(A)]$$

$$P(B \text{ hits}) = \frac{2}{5} \quad [P(B)]$$

$$P(A \text{ doesn't hit}) = 1 - \frac{1}{3} = \frac{2}{3} \quad [P(A) + P(A') = 1]$$

$$P(A') = \frac{2}{3}$$

~~$$P(B \text{ doesn't hit}) = 1 - \frac{2}{5} = \frac{3}{5}$$~~

$$P(B') = \frac{3}{5}$$

As these are independent events = $P(\text{not hitting})$
 $= P(A') \cdot P(B')$

$$[P(A \cap B) = P(A) \cdot P(B)]$$



Q. 8.

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$$P(\text{target is hit}) = 1 - P(\text{target not hit})$$

$$= 1 - P(A') \times P(B')$$

$$= 1 - \frac{2}{3} \times \frac{3}{5}$$

$$= 1 - \frac{2}{5}$$

$$= \frac{3}{5}$$

Answer: Probability of target being hit
is $\frac{3}{5}$.

Question 5.

$$\text{Plane P} \Rightarrow x - y + z + \lambda = 0$$

$$\text{point Q} = (1, 1, 1) = (x_1, y_1, z_1)$$

$$\text{Dist. of point from plane} = \left| \frac{x_1 - y_1 + z_1 + \lambda}{\sqrt{(1)^2 + (-1)^2 + (1)^2}} \right|$$

$$\frac{5}{\sqrt{3}} = \left| \frac{1 - 1 + 1 + \lambda}{\sqrt{3}} \right|$$

$$\frac{5}{\sqrt{3}} = \left| \frac{1 + \lambda}{\sqrt{3}} \right|$$

$$\frac{5}{\sqrt{3}} = \frac{1 + \lambda}{\sqrt{3}}$$

$$\text{OR } \frac{5}{\sqrt{3}} = -\left(\frac{1 + \lambda}{\sqrt{3}}\right)$$

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$$S = 1 + \lambda \quad \text{or} \quad S = -(1 + \lambda)$$

$$\lambda = 4 \quad \text{or} \quad \lambda = -6$$

Answer: $\lambda = 4, -6$

Question: 6.

$$\int \frac{dx}{x^2 - 6x + 13}$$

$$= \int \frac{dx}{x^2 - 6x + 9 + 4}$$

$$= \int \frac{dx}{(x-3)^2 + 2^2} \quad \left[x^2 - 2ax + a^2 = (x-a)^2 \right]$$

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$$= \frac{1}{2} \tan^{-1} \left(\frac{x-3}{2} \right) + C \quad [\text{where } C \text{ is constant}]$$

$$\left[\int \frac{1}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} \right]$$

Answer: $\frac{1}{2} \tan^{-1} \left(\frac{x-3}{2} \right) + C$



Section : B

Question: 7

(choice-2)

$$|\vec{a}| = 3$$

$$|\vec{b}| = 5$$

$$|\vec{c}| = 4$$

$$\vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$$(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = \vec{0} \cdot \vec{0}$$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2)}{2}$$

$$= -\frac{(3^2 + 5^2 + 4^2)}{2}$$



$$= -\left(\frac{50}{2}\right) = -25$$

$$\text{Answer: } \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -25$$

Question : 8

$$I = \int_{-1}^2 |x^3 - x| dx$$

$$I = \int_{-1}^2 |x(x-1)(x+1)|$$

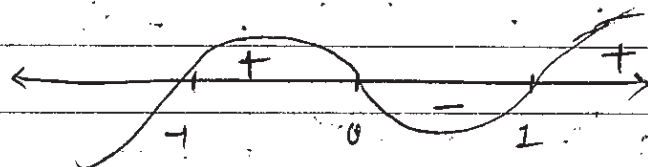
It changes sign at 0, 1, -1. So, we have to break limits here.

$$I = \int_{-1}^0 |x^3 - x| dx + \int_0^1 |x^3 - x| dx + \int_1^2 |x^3 - x| dx$$

$$\left[\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \right] \quad a < c < b$$

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In $(-1, 0)$ $x^3 - x$ is +ve

In $(0, 1)$ $x^3 - x$ is -ve

In $(1, 2)$ $x^3 - x$ is +ve

$$I = \int_{-1}^0 (x^3 - x) dx + \int_0^1 -(x^3 - x) dx + \int_1^2 (x^3 - x) dx$$

$$I = \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^0 + \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_0^1 + \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_1^2$$

$$I = \left(0 - \left(\frac{1}{4} - \frac{1}{2} \right) \right) - \left(\left(\frac{1}{4} - \frac{1}{2} \right) - 0 \right) + \left(\left(\frac{16}{4} - \frac{4}{2} \right) - \left(\frac{1}{4} - \frac{1}{2} \right) \right)$$

$$I = \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4}$$

$$I = 2 + \frac{3}{4}$$

$$I = \frac{11}{4}$$

Answer: $\int_{-1}^2 |x^3 - x| dx = \frac{11}{4}$

Question : 9.

Two lines are coplanar if they are parallel or intersecting.

$$\frac{x-1}{2} = \frac{y-3}{4} = \frac{z}{-1} \quad \text{and} \quad \frac{x-4}{3} = \frac{y-2}{-4} = \frac{z+1}{1}$$

converting to standard form,

$$\frac{x+1}{-2} = \frac{y-3}{4} = \frac{z}{-1} \quad \text{and} \quad \frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{1}$$



clearly lines are not parallel as dirⁿ ratios are not proportional.

Now we can check whether they are intersecting or not.

$$x_1 = 1$$

$$y_1 = 3$$

$$z_1 = 0$$

$$x_2 = 4$$

$$y_2 = 1$$

$$z_2 = 1$$

$$a_1 = -2$$

$$b_1 = 4$$

$$c_1 = -1$$

$$a_2 = 3$$

$$b_2 = -2$$

$$c_2 = 1$$

For lines to be intersecting, shortest distance should be zero (0).

For this, the following determinant should be zero (0).

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$$\begin{array}{c|ccc} & x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ \hline & a_1 & b_1 & c_1 \\ & a_2 & b_2 & c_2 \end{array}$$

$$\begin{array}{c|ccc} & -4 - (-1) & (-1) - (-3) & (-1) - 0 \\ \hline & -2 & -4 & -1 \\ & 3 & -2 & -1 \end{array}$$

$$\begin{array}{c|ccc} & -3 & 2 & -1 \\ \hline & -2 & -4 & -1 \\ & 3 & -2 & -1 \end{array}$$

$$= -3((-4)(-1) - (-2)(-1)) - 2((-2)(-1) - (3)(-1)) + (-1)((-2)(-2) - (3)(-4))$$

$$= -3(4 - 2) - 2(2 + 3) - 1(4 + 12)$$

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$$\Delta = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 4-1 & 1-3 & 1-0 \\ -2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix}$$

$$\Delta = 3(4-2) - (-2)(-2+3) + 1(4-12)$$

$$\Delta = 3 \times 2 + 2 \times 1 + 1 \times (-8)$$

$$\Delta = 6 + 2 - 8$$

$$\Delta = 0$$

Thus, these lines are intersecting.

Hence proved, they are coplanar.

Question: 10.

(choice-2)

$$x \frac{dy}{dx} = y (\log y - \log x + 1)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} \left(\log \left(\frac{y}{x} \right) + 1 \right) \quad \left[\log a - \log b = \log \left(\frac{a}{b} \right) \right]$$

on putting $x = \lambda x$, $y = \lambda y$.

$$f(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x} \left(\log \left(\frac{\lambda y}{\lambda x} \right) + 1 \right)$$

$$f(\lambda x, \lambda y) = \frac{y}{x} \left(\log \left(\frac{y}{x} \right) + 1 \right)$$

$$= f(x, y)$$

Thus, this equation is homogenous equation.



Let $\frac{y}{x} = t$ or $y = tx$

on differentiating with respect to x

$$\frac{dy}{dx} = t + x \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{y}{x} \left(\log\left(\frac{y}{x}\right) + 1 \right)$$

$$t + x \frac{dt}{dx} = t (\log t + 1) \quad \left[\frac{y}{x} = t \right]$$

$$t + x \frac{dt}{dx} = t \log t + t$$

$$x \frac{dt}{dx} = t \log t$$

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$$\frac{dt}{t \log t} = \frac{dx}{x}$$

on integrating both sides.

$$\int \frac{dt}{t \log t} = \int \frac{dx}{x}$$

$$\int \frac{dt}{t \log t} = \log_e \log x + c$$

let $\log t = u$

on differentiating,

$$\frac{1}{t} dt = du$$

$$\int \frac{du}{u} = \log_e x + c \quad [\log_e x = \ln x]$$



$$\ln u = \ln x + c \quad [c \text{ is integration constant}]$$

$$\ln(\ln t) = \ln x + c \quad [u = \log_e t = \ln t]$$

$$\ln(\ln(y/x)) = \ln x + c$$

$$\ln(\ln(y/x)) - \ln x = c$$

$$\ln\left(\frac{\ln(y/x)}{x}\right) = c \quad \checkmark$$

$$[\log a - \log b = \log(a/b)]$$

Answer: $\ln\left(\frac{\ln(y/x)}{x}\right) = c$

$$[\text{where } \ln x = \log_e x]$$



Question: 11

(choice - 1) .

$$I = \int \frac{x^2}{(x^2+1)(3x^2+4)} dx$$

Assuming $x^2 = y$ for simplicity.

$$\frac{y}{(y+1)(3y+4)} = \frac{A}{y+1} + \frac{B}{3y+4}$$

$$y = A(3y+4) + B(y+1)$$

on equating coefficients,

$$3A + B = 1$$

$$4A + B = 0$$

on solving we get $A = -1, B = 4$



$$I = \int \frac{-1}{x^2+1} + \int \frac{4}{3x^2+4}$$

$$I = \frac{-1}{1} \tan^{-1}\left(\frac{x}{1}\right) + \frac{4}{3} \int \frac{1}{x^2 + 4/3}$$

$$I = -\tan^{-1}x + \frac{4}{3} \int \frac{1}{x^2 + (\sqrt{4/3})^2}$$

$$I = -\tan^{-1}x + \frac{4}{3} \left[\int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) \right]$$

$$I = -\tan^{-1}x + \frac{4/3}{\sqrt{4/3}} \tan^{-1}\left(\frac{x}{\sqrt{4/3}}\right) + C$$

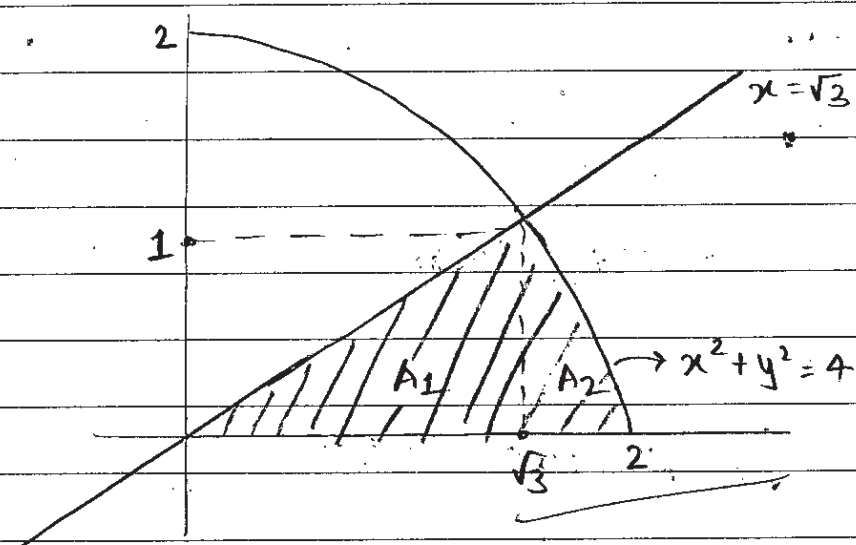
$$I = -\tan^{-1}x + \sqrt{\frac{4}{3}} \tan^{-1}\left(\frac{x\sqrt{3}}{2}\right) + C$$



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Answer : $\frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{x\sqrt{3}}{2} \right) - \tan^{-1} x + C$

Question : 12



Area enclosed = $A_1 + A_2$





$$A_1 = \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}} dx \quad [\text{line } y = \sqrt{3}x]$$

$$A_2 = \int_{\sqrt{3}}^2 \sqrt{4 - x^2} dx \quad [\text{circle } x^2 + y^2 = a^2]$$

$$A_1 = \left[\frac{x^2}{2\sqrt{3}} \right]_0^{\sqrt{3}}$$

$$A_1 = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \text{ sq. units}$$

$$A_2 = \int_{\sqrt{3}}^2 \sqrt{2^2 - x^2} dx$$

$$A_2 = \left[\frac{x}{2} \sqrt{2^2 - x^2} + \frac{2^2}{2} \sin^{-1} \left(\frac{x}{2} \right) \right]_{\sqrt{3}}^2$$



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$$\left[\int \sqrt{a^2 - x^2} = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) \right]$$

$$A_2 = \left[\left(0 + 2 \sin^{-1} 1 \right) - \left(\frac{\sqrt{3}}{2} + 2 \sin^{-1} \left(\frac{\sqrt{3}}{2} \right) \right) \right]$$

$$A_2 = 2 \left(\frac{\pi}{2} \right) - \frac{\sqrt{3}}{2} - 2 \left(\frac{\pi}{3} \right)$$

$$A_2 = \frac{\pi}{3} - \frac{\sqrt{3}}{2}$$

$$\text{Area enclosed} = A_1 + A_2$$

$$= \frac{\sqrt{3}}{2} + \frac{\pi}{3} - \frac{\sqrt{3}}{2}$$

$$\text{Area} = \frac{\pi}{3} \text{ square units}$$

Answer: Area enclosed is $\frac{11}{3}$ (1.05) square units.

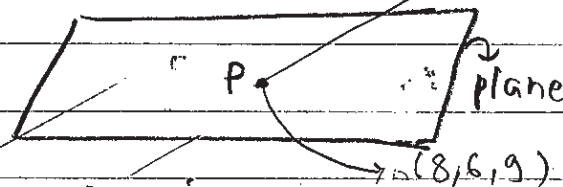
Question: 13

Line $\vec{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$

Let P be any point on this line $\vec{r}$,

~~Let P be any point on this line $\vec{r}$,~~

$O = (1, -2, 9)$



Let P be intersection of line $\vec{r}$ with the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10$.

$P = (4 + 3\lambda, 2 + 4\lambda, 7 + 2\lambda)$

Coordinates from line $\vec{r}$.

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P should also satisfy the eqⁿ of plane

$$\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10$$

$$x - y + z = 10$$

on satisfying P in this equation.

$$4 + 3\lambda - (2 + 4\lambda) + 7 + 2\lambda = 10$$

$$9 + \lambda = 10$$

$$\lambda = 1$$

coordinates of P = ~~(4, 3, 2)~~ $(4 + 3\lambda, 2 + 4\lambda, 7 + 2\lambda)$

$$= (7, 6, 9)$$



Distance of P from $(1, -2, 9)$

$$= \sqrt{(7-1)^2 + (6-(-2))^2 + (9-9)^2}$$

$$= \sqrt{6^2 + 8^2 + 0^2}$$

$$= \sqrt{64 + 64}$$

$$= \sqrt{128}$$

Distance = 10 units

Answer: distance from point of intersection
is 10 units.

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Question : 14

E be the event that seed germinates.

$$(a) \quad P(E) = P(A_1) \cdot P(E/A_1) + P(A_2) \cdot P(E/A_2) + P(A_3) \cdot P(E/A_3)$$

$$P(A_1) = 4/(4+4+2) = 4/10 \quad P(A_2) = 4/10 \quad P(A_3) = 2/10$$

$A_1 \rightarrow$ event of choosing flower seed A_1

$A_2 \rightarrow$ event of choosing flower seed A_2

$A_3 \rightarrow$ event of choosing flower seed A_3 .

$E/A_1 \rightarrow$ event E occurring given A_1 has occurred.
similarly E/A_2 and E/A_3 .

$$\begin{aligned} P(E) &= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100} \\ &= \frac{180 + 240 + 70}{1000} \end{aligned}$$

$$P(E) = \frac{490}{1000} = 0.49$$

$$(b) \quad P(A_2/E) = \frac{P(A_2) \cdot P(E/A_2)}{P(A_1)P(E/A_1) + P(A_2)P(E/A_2) + P(A_3)P(E/A_3)}$$

[Baye's Theorem]

$$= \frac{\frac{4}{10} \times \frac{60}{100}}{\frac{4}{10} \times \frac{60}{100} + \frac{4}{10} \times \frac{40}{100} + \frac{1}{10} \times \frac{60}{100}}$$

$$= \frac{490}{1000}$$

$$= \frac{49}{100}$$

$$= \frac{24}{49}$$

Answer:

$$a \Rightarrow 0.49 \text{ or } \frac{49}{100}$$

$$b \Rightarrow \frac{24}{49}$$

